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The Effectiveness of Empirical Analysis of Digital Finance based on Logistic Regression Model and the Measurement of Empirical Model Risk

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Abstract

Digital finance represents the trend of financial development in the 21st century, with its own unique basic concepts and theoretical foundations, and is currently a hot research field in the academic community. In the process of constructing the theory of digital finance, a large number of research literature has been used to conduct empirical studies on various topics, with the help of the Digital Inclusive Finance Index developed by the Digital Finance Research Center of Peking University, which is a convenient definition of digital finance. However, the validity of many empirical research results is still worth further investigation. This article has conducted a comprehensive study on this point, defining new concepts for the first time and building a new analytical framework. Through logistic regression model, 1137 empirical results were empirically studied, and it was found that less than 30% of the empirical models met the criteria of low risk and could be adopted. The new achievement of this article is the first definition of the empirical model risk level index, which provides a specific expression of the interval estimation of the model fitting risk level index, namely the calculation formulas for the upper and lower bounds of the interval. The Shannon's information entropy is applied to supplement the reliability test of the empirical results of the regression model and further conduct variance tests. Therefore, in order to avoid modern financial risks, especially financial operational risks, constructing empirical models strictly according to the testing standards in this article is highly likely to resolve the occurrence of financial operational risks.

Keywords: Digital finance, Empirical model risk level index, Financial operational risk, Information entropy, Logistic function, Validity of empirical model.

1. Introduction and Literature Review

Compared to traditional financial theory, digital finance represents the development trend of finance in the 21st century and is a thriving new financial discipline. So far, there is no authoritative definition in academia on how to define the concept of digital finance. There are at least several definitions of digital finance that can be found in literature.

(a) In the era of financial digitization, such as the Global Interbank Financial Telecommunication Society (SWIFT) established in Belgium in 1973, its biggest feature is the separation of instant payment and clearing, and the earliest online payment tool PayPal was born in the United States in 1998 (S Wang, 2022); Alipay launched in 2004 (J. Zou, Y. Zhang, 2021); Yu'ebao, launched in 2013 (Y.P. Huang, 2018).

(b) In the digital age of currency, for example, Satoshi Nakamoto first publicly released the Bitcoin white paper Bitcoin on January 3, 2008 (Q. TANG， et al., 2022); Digital Renminbi (Z. Shen. 2022).

(c) The online era of financial transactions, such as Internet finance based on financial technology (H.Huang. 2018).

(d) The exponential form of digital finance, such as the Digital Inclusive Finance Index (F. Guo, et al.,2020).

Although there is no universal consensus in the academic community on the definition of digital finance, this does not hinder the rich and diverse research activities carried out by the academic community on this hot topic. In recent years, a large number of research articles on digital finance have been published in various academic journals, gradually forming one of the distinctive hot research fields in social sciences.

Based on academic articles published in recent years, there are several research directions that can be roughly summarized.

Digital finance not only significantly affects the utilization efficiency of new energy (K.L. Wang, B.Zhao, 2021; S. Wang, 2023a), weakens speculation in the real estate market (R. Zhang, Y. Pang, 2024), interprets the prevention and control effect of China during the epidemic of COVID-19 (S. Wang, 2023b), but also plays an increasingly important role in technological innovation, industrial development and economic development.

From both national and platform perspectives, digital finance has significantly influenced the relationship between fundraising and investor numbers (S. Estrin, et al., 2024), improving the problem of low investment efficiency in companies (A. Nisar, et al., 2024), increasing research and development investment levels, promoting green technology innovation (H. Sun, 2024), accelerating innovation in green technologies, and thereby reducing carbon emissions (H. Sun, 2022; Y. Lu, 2024), enhancing the resilience of economic growth (W.W. Zhang, 2024). Digital finance plays a positive role in promoting technology diffusion (X.J. Tan, 2024), with significant spatial spillover effects (X.H. Nie, Q. Wu, 2021).

From the perspective of economic development at the provincial, municipal, and even national levels, digital finance has promoted inclusive economic growth in China (X Zhang, et al., 2019; H.Z. Qian, et al., 2020; L.Y.Sun, et al., 2021), and has a significant positive impact on the practice of the real economy, including technological innovation (G.S. HE, X. LIU, 2024), promoted industrial structure upgrading (R. Zhao, et al., 2021), and has a significant promoting effect on the development of industries, especially the second and third industries (G.R. Xia, 2021; M.X. Lin, Y.B. Xiao, 2022). In 2022, it will significantly promote industrial development, especially in the eastern region (C.J. Huang, Y.C. Wen, 2024), improve the innovation level of cities (S. Pan, et al., 2021). Digital finance promotes the improvement of urban innovation capability by promoting capital agglomeration and high-quality labor agglomeration (X.F. Wu, et al., 2022), affecting the flow of labor force population (S.Z. Ma, Z.X. Hu, 2022).

Although digital finance has deepened the degree of multidimensional poverty through the digital divide (Z.Y. He, et al., 2020), it has significantly reduced the relative poverty level (J.G. Sun, et al., 2020), thereby playing a critical role in the economic development and innovation of rural areas (L. Teng, et al., 2021; T.Y. Yang, A.J. Wang, 2022), promoting the development of digital education for farmers and the performance of rural enterprises (T. Wen, Y.B. Liu, 2023; Y.R. Chen, et al., 2024), and encouraging household consumption growth (Y.K. Wang, Y.W., Wang, 2024), but unexpectedly led to an increase in household debt risk (H.J. Wang, H. Yang, 2022).

Digital finance not only helps to reduce audit fees for commercial banks (J.M. Chen, et al., 2024), but also promotes the development of banks (Z. Fu, H. Wang, 2021). However, digital finance has intensified competition among banks (K. Yuan, D.T. Zeng, 2021), affected the formulation of bank loan contracts (C. Liu, 2022), and weakened the effectiveness of monetary policy (J. He, T. Wei, 2022). However, the development of digital finance has a actively positive effect on the stability of finance (D.B. Yao, D. Wu, 2022).

Digital finance not only constructively improves the risk resistance of non-financial enterprises (F.L. Yan, et al., 2021), but also has a positive impact on the financing constraints of small and medium-sized enterprises (Y. Bu, et al., 2024), This is reflected in at least four aspects, namely, firstly, alleviating financing constraints for small and medium-sized enterprises by reducing financing costs and improving financing structures (X.S. Xie, S.P. Yan, 2022); secondly, alleviating financing constraints for enterprises through market mechanisms and government regulation (Y.K. Li, X.K. Liu, 2022); thirdly, alleviating financing constraints through supply chain finance channels (H.F. Jiang, Y.P. Liu, 2021); fourthly, alleviating financing constraints for enterprises by improving financing structures, marketization levels, and reducing financing costs and leverage levels (R. Huang, et al., 2021); and thirdly, weakening the leverage manipulation ability of enterprises from two dimensions of financing constraints and information transparency in digital finance (B. Yu, et al., 2024). Relieving the financing constraints of enterprises can serve as a driving force for high-quality development (L. Teng, D.G. Ma, 2020). The development of digital finance will significantly alleviate the financing constraints of enterprises, and the relaxation of financing constraints will have a significant positive impact on enterprise innovation (J.Y. Wan, et al., 2020; X. Gao, et al., 2022; Y.X. Zheng, et al., 2022). The overall innovation effect of digital finance on enterprises is positive (J.S. Jia, Y.T. Liu, 2021), by improving the total factor productivity of enterprises, to achieve high-quality economic development (H.L. Jiang, P.C. Jiang, 2021). Technological innovation drives economic growth (S. Tang, et al., 2020; W.C. Xu, A.J. Fan, 2022) to achieve an increase in the income level of workers (H. Ai, X.M. Huang, 2023) and promote the prosperity of household consumption (Z.X. Wang, C.X. Wang, 2024). Investment is one of the main driving factors of economic growth. The impact of digital finance on the adjustment of corporate capital structure is positive and significant (Z.Q. Wang, et al., 2023), strengthening corporate investment behavior by weakening financing constraints (P.C. Jiang, H.L. Jiang, 2023), and intensifying the competitive situation of corporate innovation (Y.J. Wang, et al., 2024). Digital finance has significantly promoted technological innovation in small and medium-sized enterprises by improving the profit environment, reducing financing costs, easing financing constraints, and enhancing financing functions (X.Y. Xie, X.Y. Zhu, 2021; Y.Q. Tao, et al., 2021). It can also promote enterprise innovation by influencing the pledge rights of controlling shareholders (Y.K. Li, et al., 2021), and further promote cross-border mergers and acquisitions by alleviating financing constraints and enhancing technological innovation (X.Y. Jin, W.F. Zhang, 2021), in order to achieve high-quality development strategies.

From the above literature review, it can be seen that the research trend of digital finance is taking shape, and its research topics are very rich. However, it has not yet reached the peak of its academic research, and there are still many basic research works that need to be gradually developed and improved in the long years to come. However, based on the research literature currently available, the main problem lies in the effectiveness of empirical models. In other words, many empirical studies are even ineffective, despite emphasizing that their research results are robust and reliable. At the same time, the financial operational risks faced by these models, which suffer from a significant lack of empirical evidence, cannot be ignored in guiding the practical work of financial management and specific financial business operations. Below, we will mainly explore these issues, provide theoretical characterization and empirical analysis, and propose targeted countermeasures for these financial risks, in order to provide necessary practical references for the academic community and practical departments.

2. Theory and Methods

In the teaching of linear regression equations in econometrics, the regression analysis conducted using the least squares method has an equation that is:

$$SST=SSR+SSE,$$

where, SST represents the total sum of squared deviations, SSR represents the regression sum of squared deviations, and SSE represents the residual sum of squared deviations. The goodness of fit value R^2 defined based on this relationship is:

$$R^2=\frac{SSR}{SST}=1-\frac{SSE}{SST}, \qquad (1)$$

The numerical value of R^2 represents the degree of fit of the model to the data or sample. The larger its value, the higher the fit of the model to the data, the stronger the explanatory power of the model, and the more optimized the model. When the value of R^2 is less than a certain value, such as 0.5, it indicates that the model's ability to interpret data is weaker, indicating that the model is no longer suitable and needs to be re optimized. If we continue to use models with goodness of fit values less than 0.5, it means that the risk of model selection cannot be ignored. If such a risky model is used to guide actual financial business work or develop corresponding financial management systems, the financial operational risks cannot be avoided. The higher the goodness of fit value, the lower the risk of selecting the corresponding model; The smaller the goodness of fit value, the greater the risk of selecting the corresponding model. In order to explore the risk situation of model use, we first provide the definition of indicators for measuring the risk of model use.

Definition 1. The risk level index simulated by a model, also known as the empirical model risk level index, refers to the inverse of the goodness of fit value obtained during empirical regression analysis based on the model.

The inverse of the goodness of fit value is used as the risk level index to measure the simulated model. The higher the risk level index of the empirical model, the higher the predicted or evaluated risk level based on the model. The model risk level index is greater than or equal to 1. The larger the risk level index of the model simulation, the worse the simulation effect of the model, and the lower the credibility of the conclusions obtained from the model simulation.

If a phenomenon is independently fitted by more than one model, its overall simulated risk level index can be defined as the reciprocal of the arithmetic mean of all goodness of fit values, i.e.:

Definition 2. The risk level index simulated by a set of models refers to the reciprocal of the average value of the corresponding goodness of fit obtained during empirical regression analysis of this set of models.

These definitions can serve as risk level measurement indicators for assessing the effectiveness of model simulations. It should be pointed out that when using indicators such as geometric mean or harmonic mean, attention must be paid to the rationality and feasibility of the definition.

The range of indicators for simulating the risk level index in a normal model should be between 1 and 2, indicating that the model has a high degree of credibility in simulating the sample, and also indicating that the model's setting or selection is highly scientific and reasonable; If the simulated risk level index of the model is greater than 2, it indicates that the simulation effect of the model is not very convincing from a statistical perspective. Its simulation of the sample is very unscientific and unreasonable, and all conclusions derived from it will be difficult to convince. The application based on such models will take great risks. The critical value for simulating the risk level index of the model is set to a certain real number, such as 2, which represents the maximum upper limit of the acceptable risk level index of the model. Risk indices exceeding 2 correspond to models that are difficult to accept.

One possible reason for the large simulated risk level index in the model is that the design of the model lacks strict scientific theoretical support, and the proportion of subjective and arbitrary factors in the selection of model components should not be small. In fact, even if there is some logical relationship between variables, a model cannot be arbitrarily arranged to describe it. The specific expression form of the model should depend on the objective requirements of the sample to be simulated, rather than being artificially assumed. From a large amount of existing research literature, there is a lack of scientific discourse in this area.

In order to evaluate the risk effectiveness of regression models, we have the following regulations.

The explanatory variable is an empirical regression model, which can be represented by the goodness of fit value R^2 estimated by the empirical model. The dependent variable is the effectiveness of the empirical regression model, which can be represented by the sign function sgn (R^2) and is a binary variable with a value of 0 or 1. Therefore, for a positive number $\eta\in(0,1)$, the definition of the sign function is as follows:

$$\mathrm{sgn}(R^2)=\begin{cases}1, R^2\geq\eta \\ 0, R^2<\eta\end{cases} \qquad (2)$$

The array form formed by the explanatory variable and the dependent variable is (1,1) or (1,0), which means that if there is only one empirical regression model and its goodness of fit is greater than or equal to η, the validity value of the empirical model is 1, denoted as (1,1). Otherwise, the validity value of the empirical model is 0, denoted as (1,0), the empirical model is invalid or has poor performance, and so on.

Definition 3. An empirical model is called validity, if its goodness of fit is not less than a positive number for example $\eta\in(0,1)$.

Each array is an element of a collection, that is, element=array. So, if an article has n empirical regression models, the maximum possible set of all elements is:

$$I_{n\times 1}=\{(n,0),(n-1,0),\cdots,(1,0),(1,1),\cdots,(n-1,1),(n,1)\}$$

The cardinality of its set $I_{n\times 1}$ is 2n arrays, which means it contains 2n elements. Regarding the same topic, if there are m articles, there can be a set of up to 2nxm possible arrays. If each article has at most n empirical regression models, the set is denoted as $I_{n\times m}$. Based on the previous analysis, the following theorem can be derived regarding the measurement of the effectiveness of empirical models.

Theorem 1(The validity of empirical models). For $I_{n\times m}$, the probability expectation of the validity of empirical regression models is determined by the logistic function.

Proof: According to $I_{n\times m}$, assuming there are m papers, each paper has at most n empirical regression models. Then, according to equation (2), there are at most 2nxm elements composed of 0 and 1. Set up

$$p_i=P\left(\mathrm{sgn}\left(R_i^2\right)=1\right), i=1, 2, \ldots, 2n\times m,$$

R^2 is the goodness of fit to its empirical regression model. According to the logistic regression model analysis as follows,

$$\mathrm{sgn}(R^2)=a+bR^2+\varepsilon,$$

where a and b are coefficients, and ε is a random perturbation term that conforms to logistic distribution, it is easy to get the following probability expectation can be obtained:

$$E(p)=\frac{1}{2n\times m}\sum_{i=1}^{2n\times m}\frac{1}{1+e^{-\hat{a}-\hat{b}R_i^2)}}, \qquad (3)$$

where, $\hat{a}$ and $\hat{b}$ are the estimated values of the logistic regression model's coefficents a and b, obtained by the Maximum Likelihood method respectively, and E (p) is the expectation of the probability sample space p_i (i=1, 2,..., 2n × m). The conclusion to be proved by the formula (3) is valid.

For the above array set $I_{n\times m}$, the logistic regression model can be applied to calculate the corresponding probability values. According to the empirical model validity theorem, an evaluation of the simulation effect of the model used in digital finance research can be obtained. Equation (3) is the specific metric formula for this evaluation.

When considering building a model, it is necessary to first analyze the statistical distribution of the data before selecting the corresponding model. If statistical distribution analysis is not conducted, it is unclear what appropriate model needs to be selected for regression. Therefore, the first step is to provide a statistical description of the data, including the calculation of its eigenvalues. Only an empirical model established in this way can maximize the rationality and robustness of financial decisions made based on the model.

The game risk that exists in the expansion of financial business is the operational risk of financial business caused by information asymmetry. The way of game theory is to obtain basic information and make judgments based on practical experience generated by a solid theoretical foundation. The deep integration of the two can form rational decision-making behavior for financial business development. In the digital age, the acquisition of game supporting information relies strictly on the level of network search engines and software program settings. This is reflected in the fact that information analysis mainly comes from the display of machine instruments, just like the technical indicators displayed in physical examinations. Allocate limited traditional manual tasks to statistical analysis conclusions based on the largest possible database. The key factors here are the breadth and depth of information search, as well as the scientific and rational nature of program design. Considering the limitations of human cognitive level, such process judgments may deviate and mislead the direction of execution, reducing the actual effectiveness of execution and the degree of conformity with the actual situation. If this happens, it means that these studies have fallen into the shackles of a 'technical trap' and will be difficult to break through in a short period of time.

The constraints of financial business games are costs (economic costs, opportunity costs, etc.). How to maintain a certain dynamic optimization solution is a financial mathematical problem worth studying.

In the era of artificial intelligence, models are the foundation for the effectiveness of artificial intelligence. If the setting of the model is not reasonable, the risks derived from its application will continue to amplify with the improvement of artificial intelligence algorithms, and the results may be disastrous, depending on the specific field in which the model is applied. The biggest concern is that the application of artificial intelligence based on these improperly designed models in the practical field of financial practice may lead to significant operational risks in the financial system, which must be prevented in advance.

Theorem 2.(Model fitting risk interval measurement). For $I_{n\times m}$, when the sample size satisfies the following relationship:

$$n\times m>\frac{Z_{\frac{\alpha}{2}}^2\sigma^2}{2E(R^2)^2}, \qquad (4)$$

then the risk-free interval or tolerable low-risk interval of the model fitting risk level index can be expressed in the following form:

$$\left(\min\left\{1,\frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}+Z_{\alpha/2}\sigma}\right\},\min\left\{\frac{1}{\eta},\frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}-Z_{\alpha/2}\sigma}\right\}\right) \qquad (5)$$

where, positive numbers $\eta\in(0,1)$, $\left\{R_i^2\right\}_{i=1}^{2n\times m}$. To fit the goodness of fit sample space, 2nxm is the sample size, σ is the standard deviation, $E(R^2)$ is the expectation, α is the significance level, and $Z_{\frac{\alpha}{2}}$ is the critical value of the standard normal distribution of the Z statistic.

Proof: According to the set $I_{n\times m}$, there are at most 2nxm cases of R_i^2 that form a goodness of fit sample space. For any given significance level α, the interval estimation for the sample space $\left\{R_i^2\right\}_{i=1}^{2n\times m}$ is as follows:

$$\left(E\left(R^2\right)-Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{2n\times m}},E\left(R^2\right)+Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{2n\times m}}\right)$$

According to condition (4) and the definition of the risk level index fitted by the model, there are

$$\frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}+Z_{\alpha/2}\sigma},\frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}-Z_{\alpha/2}\sigma}$$

In the sample space composed of goodness of fit, the interval estimate of the model fitting risk level index can be obtained through the Z-statistic as follows:

$$\left(\frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}+Z_{\alpha/2}\sigma}, \frac{\sqrt{2n\times m}}{E(R^2)\sqrt{2n\times m}-Z_{\alpha/2}\sigma}\right) \qquad (6)$$

Considering the effectiveness of the empirical model, we will now validate the equation by assuming that:

$$\min\left\{1, \frac{\sqrt{2n\times m}}{E\left(R^2\right)\sqrt{2n\times m}+Z_{\alpha/2}\sigma}\right\}, \min\left\{\frac{1}{\eta}, \frac{\sqrt{2n\times m}}{E\left(R^2\right)\sqrt{2n\times m}-Z_{\alpha/2}\sigma}\right\},$$

Combined with equation (6), it can be concluded that equation (5) is correct.

Note: Equation (5) is the result of performing an effective transformation, otherwise, the model fitting of the risk level index interval may be invalid. This is absolutely necessary for assessing the effectiveness of empirical regression models.

For $I_{n\times m}$, when the model settings are reasonable, its simulated risk level index should fall within the interval of equation (5), and the resulting information entropy should be zero. It can be inferred that the smaller the information entropy, the more reliable the empirical results of the model regression. According to the validity theorem of empirical models and the definition of information entropy by Shannon, the formula for calculating information entropy in regression models is as follows:

$$\text{Empirical regression model information entropy} = -\sum_{i=1}^{2n\times m}\frac{1}{1+e^{-\hat{a}-\hat{b}R_i^2)}}\ln\frac{1}{1+e^{-\hat{a}-\hat{b}R_i^2)}}, \qquad (7)$$

where $2n \times m$ is the sample size, parameters $\hat{a}$ and $\hat{b}$ are estimated values of the logistic regression model parameters obtained by maximum likelihood method, and R_i^2 is the goodness of fit value of the i-th regression model. The larger the information entropy, the higher the risk level index of the empirical analysis of the model, and the greater the operational risk taken based on the empirical results of the model.

3. Empirical Analysis

The expectations and variances used in theoretical proof are replaced by the mean and variance of the sample in empirical testing, which does not exceed the norms of theory.

The data collection source is the parameters obtained from the regression empirical models mentioned in the references, and the sample consists of 1137 parameters obtained from empirical models. Each sample point corresponds to an empirical regression model, although different articles may use the same or similar model. The model may exhibit repetitive phenomena, but all the objects studied by the models are independent of each other, which ensures that there is no real correlation between the samples collected in this study. Therefore, this study is effective. Due to the processing of the collected data through sign functions in this article, the dependent variable is a binary variable, and the values obtained for the explanatory variable are independent sets derived from 1137 empirical models, totaling 1137 sample data. Based on the structural characteristics of the sample, this article applies the logistic regression model for empirical research. If η=0.5, the regression report obtained through Eviews software is as follows:

Coefficient covariance computed using observed Hessian				
Variable	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
C	-51.21517	10.06618	-5.087848	0.0000
R^2	101.6106	20.11034	5.052657	0.0000
McFadden R-squared	0.958488	Mean dependent var		0.268722
S.D. dependent var	0.443491	S.E. of regression		0.063783
Akaike info criterion	0.051843	Sum squared resid		4.609421
Schwarz criterion	0.060715	Log likelihood		-27.42111
Hannan-Quinn criter.	0.055194	Deviance		54.84222
Restr. deviance	1321.104	Restr. log likelihood		-660.5519
LR statistic	1266.262	Avg. log likelihood		-0.024160
Prob(LR statistic)	0.000000			
Obs with Dep=0	830	Total obs.		1135
Obs with Dep=1	305			

According to the regression report guidance, the goodness of fit values provided by the sample significantly explain less than 30% of the reliability of the model corresponding to the sample. Based on the analysis results of this regression model, it can be inferred that many empirical models established in published articles are very ineffective and strictly speaking unreasonable. Further discussion will be discussed below. The interval for fitting the risk level index of the model obtained from the sample is estimated to be at the significant level of α=5%

$$(2.7535, 3.0241).$$

According to the model fitting risk interval measurement theorem, the risk-free interval or tolerable low-risk interval for the model fitting risk level index should be (1, 2), and the probability of obtaining the true value is only 2.5%. The empirical results obtained from the logistic regression model are reliable.

4. Results and Discussion

In order to further verify the robustness of the above results, this article calculated the dispersion coefficient of goodness of fit based on the sample: 0.805, indicating that the distribution of goodness of fit values is compact, and most of the goodness of fit values are clustered around their mean values. In order to explore the average level of these goodness of fit values, the regression report obtained through analysis of variance is as follows:

Dependent Variable: R^2 Method: Least Squares Sample: 1 1137 Included observations: 1137				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	0.198919	0.004674	42.55736	0.0000
$\text{sgn}(R^2)$	0.551428	0.009010	61.20236	0.0000
R-squared	0.767453	Mean dependent var		0.347324
Adjusted R-squared	0.767248	S.D. dependent var		0.279290
S.E. of regression	0.134742	Akaike info criterion		-1.169159
Sum squared resid	20.60625	Schwarz criterion		-1.160300
Log likelihood	666.6667	Hannan-Quinn criter.		-1.165813
F-statistic	3745.729	Durbin-Watson stat		0.996050
Prob(F-statistic)	0.000000			

The above analysis of variance report significantly explains that the analysis of goodness of fit is appropriate. According to the regression report, the calculated average goodness of fit value is:

$$\overline{R^2} = 0.3473.$$

This average goodness of fit indicates that the effectiveness of regression analysis occurs in the region with a probability of 2.5%, which is the second type of statistical error, the false error. This result explains a serious academic ecological fact: over 75% of empirical regression analysis results are invalid but mistakenly considered valid.

In fact, based on the 1137 R^2 values obtained from the analysis of variance, connected by broken line segments, the position indicated by the red dashed line segment in Figure 1 is the average goodness of fit obtained from the variance regression analysis. The rectangle marked by the green line is located at 50% of the goodness of fit. The empirical regression model corresponding to the goodness of fit value above the rectangle is valid, while the empirical model corresponding to the goodness of fit value below the rectangle is invalid. Obviously, the empirical models corresponding to the majority of goodness of fit values reflected in Figure 1 lack effectiveness. This is consistent with the results obtained from the logistic regression model.

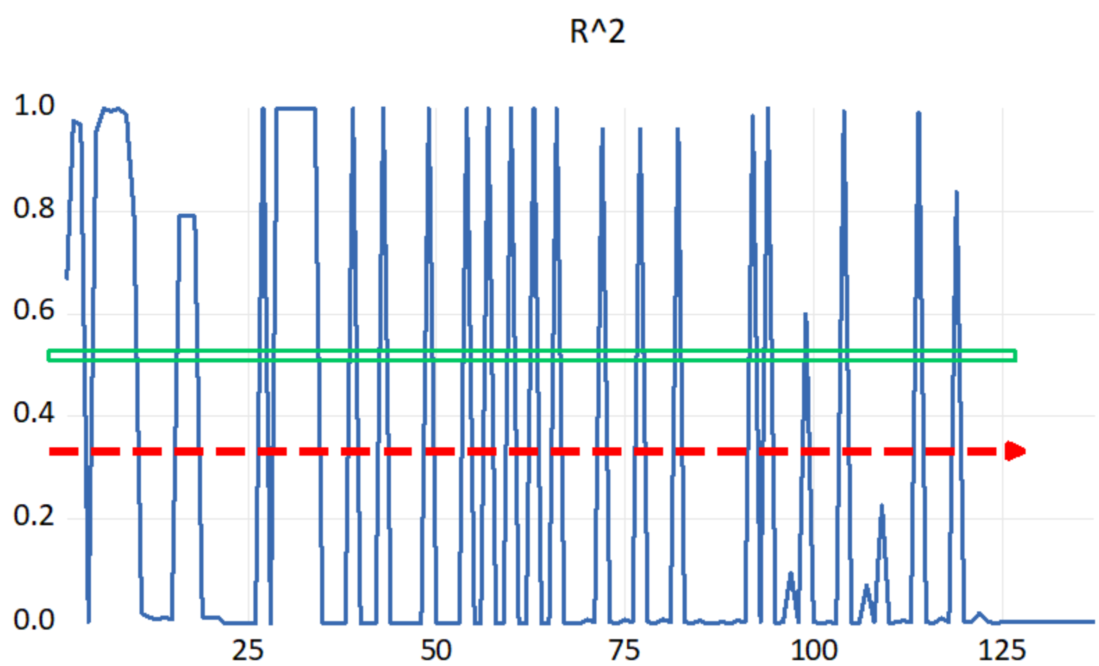

Figure 1. Line graph of goodness of fit values.

The error term U obtained from the analysis of variance has a right skewed low kurtosis normal distribution, which can be seen from the following scatter plot:

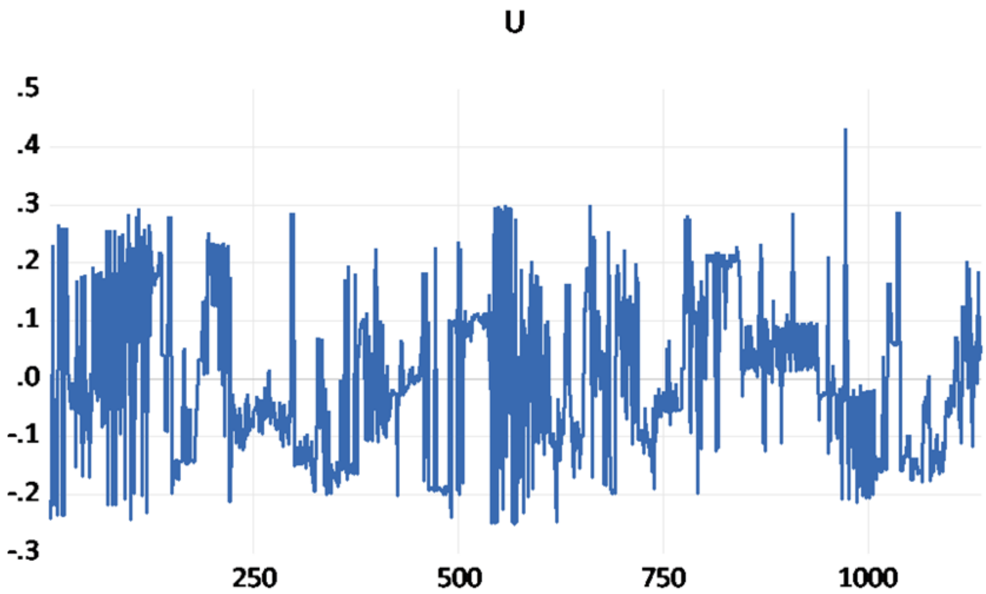

Figure 2. Scatter plot of error term in analysis of variance.
Source: Error data generated by variance analysis based on the sample.

Figure 2 shows that there is no autocorrelation in this random error term, indicating that the results of the analysis of variance are robust.

The complex situation where the effectiveness of empirical regression analysis is not evident can be supported by the information entropy of empirical regression models. According to the logistic regression model, the calculated p-ui, i=1, 2,..., 1137, its information entropy is:

The information entropy of the empirical regression model is : $-\sum_{i=1}^{1137} p_i \ln p_i = 14.76$

These two results further prove that the empirical results of a large number of regression models are unreliable.

Therefore, the results of the empirical analysis above, namely the logistic regression model regression and variance analysis, indicate that the empirical results of digital finance obtained from the references listed in the literature, overall, cannot guarantee the validity of the results claimed by the relevant literature, as the proportion of formally effective empirical models is less than 30%. This is where the risks and hidden dangers of empirical models lie, and their application likely implies that operational risks cannot be avoided. All these empirical models are widely applied in the financial field, and the policy proposals derived from them are full of uncontrollable operational risks.

In fact, in the field of finance, General Secretary Xi Jinping, the President of China, pointed out at the opening ceremony of the special seminar on promoting high-quality financial development for provincial and ministerial level leading cadres that "we must adhere to risk prevention and control as the eternal theme of financial work, and focus on preventing and resolving financial risks, especially systemic financial risks", and emphasized the establishment of a sound "complete and effective financial regulatory system". Current financial risks, such as systemic financial risk, exchange rate risk, compliance risk, investment risk, credit risk, market risk, operational risk, liquidity risk, interest rate risk, sovereign risk, strategic risk, debt crisis, liquidity crisis, currency crisis, etc., will ultimately lead to local or global liquidity failure. From the perspective of financial practice and institutional construction, if the empirical model risk level index relied upon by regulatory functional departments is greater than 2, then their process of regulating financial risks has become a tool for creating financial risks. In fact, the financial risks arising from policy formulation and institutional arrangements derived from the simulation results of the model are subtle, as the confusing conclusions disguised as "science" are also difficult to identify and eliminate in a short period of time.

According to our analysis using logistic regression models, over 70% of published empirical research conclusions on digital financial entities are highly unreliable in terms of their validity. The reliability of the conclusions obtained in this article is further supported by information entropy and variance analysis. In such an academic context, if someone or institution guides financial work based on the empirical models they study, the financial system risks faced by the implementation plans of financial affairs guided by these similar or identical empirical tools are likely unavoidable.

To do a good job, one must first sharpen their tools. Only by systematically correcting the arbitrary use of empirical models to analyze problems and guide work, returning to strict scientific research norms, and carefully using empirical model tools in practical work, can we avoid the potential harm of systemic financial risks caused by the misuse of financial instruments at present. The reason for the aggregation of risks in empirical models is not only due to the conceptual deviation of econometric models in practical applications, but also the lack of strong theoretical support for digital finance as a product of digital financial technology applications.

The current research status of digital finance is characterized by a prominent emphasis on empirical evidence over theoretical research (B. Wang, et al., 2024). The current research on digital finance mainly reflects on the level of information diffusion, which is due to the digital expansion process brought about by the advancement of digital technology. At present, many researches on digital finance mainly combine traditional financial activities with the latest digital technologies such as Internet technology, blockchain technology and big data technology to form the application characteristics of digital technology in the financial field, which is mainly reflected in the technical level. In fact, digital finance theory is not a simple composite theory of digital technology and traditional financial theory, but a new financial theory system with self consistent or semi consistent characteristics. This is a new stage in the historical development of traditional financial theory, with distinct characteristics of the 21st century. Digital finance theory can be divided into narrow digital finance theory and broad digital finance theory. The narrow definition of digital finance theory studies the dynamic linkage between the closed-loop jumping motion within the digital finance system and the projection radiation of economic activities related to digital finance. The generalized digital finance theory is a theory that studies the dynamic relationship between the jumping motion of digital finance single chain or multi chain linkage and the projection radiation of economic activities associated with digital finance. （S. Wang，2023a）。 The latest digital finance theory is still in the initial stage of its development history, and there are still many fundamental works waiting for further research and practical work in the future to enrich and deepen it. Digital finance theory is a financial theory of the 21st century, which is a new financial theory that has been developed to keep up with the times and serve as a bridge between traditional financial theories of the 20th century and earlier. The theory of digital finance will undoubtedly support the entire international community's journey towards a new human social history, and play a fundamental social pillar force for achieving a harmonious world pattern of great harmony for mankind!

5. Conclusion

The main conclusions drawn from this article are briefly stated as follows:

The first, it is the first definition of the risk level measurement index for the model simulation effect of empirical models, namely the model simulation risk level index. In today's academic community, due to holding illogical academic thinking, the establishment of empirical models is seriously disconnected from the requirements of data samples, leading to a systematic and rigid empirical research model that connects the two together. As a result, the scientific and effective nature of model establishment is intentionally or unintentionally ignored. This is the fundamental reason for the emergence of empirical model risks. This article provides specific methods for measuring empirical model risks, which are the main means of avoiding empirical model risks;

The second, it is to effectively test the risk of empirical models, and the determination of sample size is a key step. Generally speaking, under large sample conditions, the effectiveness of empirical model risk testing is guaranteed based on statistical theory. But what is the lower bound for such a large sample? Finding the answer is a very meaningful scientific research task. If the sample size is smaller than the minimum lower bound required to test the risk of the empirical model, then according to statistical theory, such a test will be invalid. Therefore, determining the minimum lower bound becomes a fundamental prerequisite that must be completed. Therefore, under the specification of the lower bound estimation formula for the required sample size, the specific manifestation of interval estimation for fitting the risk level index of the model is given, namely the calculation formula for the upper and lower bounds of the interval and the selection of effective interval modes. From a scientific perspective, based on statistical significance, an interval estimation for testing the risk of empirical models has been established, which has important scientific value;

The third, two fundamental theorems were proposed and proven, namely the empirical model validity theorem and the model fitting risk interval measurement theorem. These two theorems are the theoretical basis for the continuation of this study;

The fourth, it is to apply the rich information entropy to supplement the reliability of the empirical results of the regression model. Information entropy describes the complexity of a problem, meaning that the larger the information entropy, the more complex the problem, and vice versa. The information entropy of empirical model risk can be described by the corresponding goodness of fit value, which provides convenience for calculating the corresponding information entropy. This information entropy actually refers to the information entropy derived from empirical model risk;

The fifth, in order to avoid modern financial risks, especially financial operational risks, constructing empirical models strictly according to the testing standards in this article is highly likely to resolve the occurrence of financial risks. In order to effectively avoid the risks of empirical models, establishing scientific ideas and concepts for constructing empirical models is the most important initial stage, followed by the issue of methods for establishing empirical models. The scientific way of thinking for establishing empirical models is to (i) study the theoretical relationships on which the problem is based, (ii) design an appropriate empirical model based on the statistical behavior characteristics of the sample, and (iii) finally determine the specific form of the model through appropriate model estimation methods and conduct relevant statistical tests. Only after completing these three basic steps, selecting the optimal model as the appropriate model for empirical testing, and deriving relevant conclusions from it, can maximize the avoidance of empirical model risk issues;

The sixth, digital finance theory can be summarized as the study of the dynamic linkage between the closed-loop jumping movement of digital finance and the projection radiation of economic activities related to digital finance in a narrow sense. The generalized digital finance theory is a theory that studies the dynamic linkage between the jumping motion of digital finance single chain or multi chain linkage and the projected radiation of economic activities associated with digital finance. This is the initial description of the historical development process of digital finance theory, and there are still many fundamental works that need to be carried out in depth in order to gradually improve this new financial theory, which is the financial theory of the 21st century, also known as digital finance theory. The theory of digital finance has broken through the limitations of traditional financial theory, filled the gaps of traditional financial theory, and almost completely achieved a self consistent financial system theory;

The seventh, the fundamental reason why the risk of empirical models in digital finance is so high is that all these related papers basically adhere to the same or similar research ideas and concepts. Firstly, the similarity of model construction modes mostly includes the establishment of benchmark models (necessary conditions), robustness testing models (necessary conditions), endogeneity testing models (optional conditions), heterogeneity testing models (optional conditions), and so on, using the same or similar methods of model establishment; Secondly, there is the separation of the model and the sample. First, an empirical model is established, then data samples are collected, and finally the two are forcefully combined, such as conducting regression analysis; Finally, we hold an indifferent attitude towards the role played by goodness of fit, that is to say, as long as there is goodness of fit, it doesn't matter the size of the goodness of fit value. It is precisely this empirical research approach that has led to a significant increase in the risk of empirical models. The magnitude of the goodness of fit value measures the core issue of whether the model fits well or poorly. Only when the goodness of fit value of the model to the sample reaches a certain level, such as not less than 50%, will the explanatory power of the model to the sample data be demonstrated, and the application of the model can maximize the avoidance of unnecessary operational risks.

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